

# Density-matrix approach to coherent transport and the measurement problem

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## Abstract

Bloch-type equations for description of coherent transport in mesoscopic systems are applied for a study of the continuous measurement process. Both the detector and the measured system are described quantum mechanically. It is shown that the Schrödinger evolution of the entire system cannot be accommodated with the measurement collapse. The latter leads to quantum jumps which can be experimentally detected.

In a recent elegant experiment Buks *et al.* [1] realized a nondestructive continuous monitoring of a quantum system in the linear superposition by using the ballistic point-contact as a detector [1]. Experiments of this type are very important for understanding of the measurement process, in particular since the quantum-mechanical behavior of the detector can be traced out. The latter allows us to investigate of whether the measurement collapse generates experimentally observed effects, which are not described by the Schrödinger equation applied to the entire system. As an example we consider continuous monitoring of

a *single* electron inside the coupled-dot (Figs. 1) [2]. The point-contact (detector), shown as a barrier, is placed near one of the dots. The barrier is connected with two reservoirs at the chemical potentials  $\mu_L$  and  $\mu_R$  respectively. Since  $\mu_L > \mu_R$ , the current  $I = eD$  flows through the point-contact, where  $D = T(\mu_L - \mu_R)/(2\pi)$  and  $T = (2\pi)^2\Omega^2\rho_L\rho_L$  is the transmission coefficient. Here  $\Omega$  is the coupling between the left and the right reservoirs and  $\rho_{L,R}$  is the corresponding density of the states. The penetrability of the point-contact (the barrier height) is modulated by the electron, oscillating inside the double-dot. When the electron occupies the left dot, the transmission coefficient is  $T_1$ . However, when the right dot is occupied, the transmission coefficient  $T_2 \ll T_1$  due to the electrostatic repulsion generated by the electron. As a result, the current  $I_2 \ll I_1$ . (We assume that  $T_2 = 0$ , so that the point contact is blocked whenever the right dot is occupied). Since the difference  $\Delta I = I_1 - I_2$  is macroscopically large, one can determine which of the dots is occupied by observing the point-contact current. Yet, the entire system can be treated quantum-mechanically.

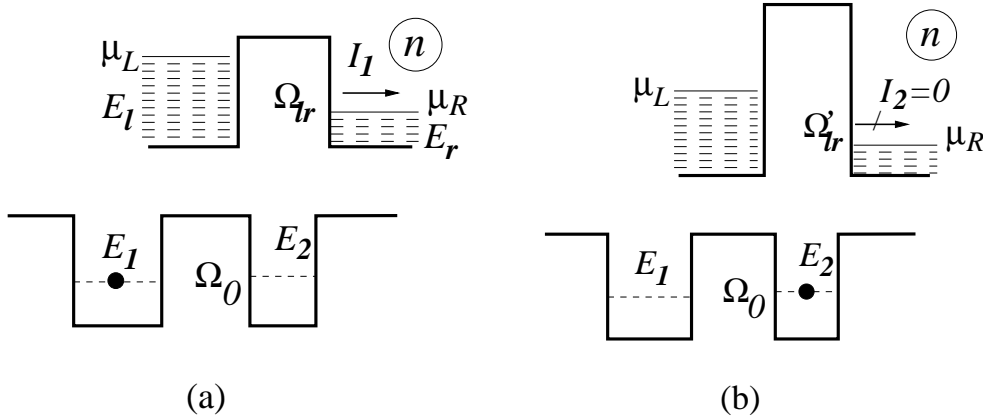


Fig. 1. The point-contact detector near the double-dot.  $\Omega_{lr}$  is the coupling between the level  $E_l$  and  $E_r$  in the left and the right reservoirs.  $\Omega_0$  is the coupling between the quantum dots. The index  $n$  denotes the number of electrons penetrating to the right reservoir (collector) at time  $t$ .

It is described by the Hamiltonian:  $\mathcal{H} = \mathcal{H}_{PC} + \mathcal{H}_{DD} + \mathcal{H}_{int}$ , where

$$\mathcal{H}_{PC} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} \Omega_{lr} (a_l^\dagger a_r + a_r^\dagger a_l), \quad (1)$$

$$\mathcal{H}_{DD} = E_1 c_1^\dagger c_1 + E_2 c_2^\dagger c_2 + \Omega_0 (c_2^\dagger c_1 + c_1^\dagger c_2), \quad (2)$$

$$\mathcal{H}_{int} = \sum_{l,r} \delta\Omega_{lr} c_2^\dagger c_2 (a_l^\dagger a_r + a_r^\dagger a_l). \quad (3)$$

Here  $\mathcal{H}_{PC}$ ,  $\mathcal{H}_{DD}$  and  $\mathcal{H}_{int}$  are the Hamiltonians describing the point-contact, double-dot and their mutual interaction, respectively. The latter affects the coupling between the reservoirs. It becomes  $\Omega'_{lr} = \Omega_{lr} + \delta\Omega_{lr}$  whenever the second dot is occupied. (In our case  $\delta\Omega_{lr} = -\Omega_{lr}$ ).

The time-development of the entire system is described by the many-body Schrödinger equation  $i\dot{\rho}(t) = [\mathcal{H}, \rho(t)]$ , where  $\rho(t)$  is the total density-matrix. It was shown [2] that the continuum reservoirs states can be integrated out in the density-matrix  $\rho$ . Then the equation of motion becomes a system of coupled Bloch-type equations for the reduced density-matrix  $\sigma(t)$ :

$$\dot{\sigma}_{11}^{(n)} = -D_1 \sigma_{11}^{(n)} + D_1 \sigma_{11}^{(n-1)} + i\Omega_0 (\sigma_{12}^{(n)} - \sigma_{21}^{(n)}), \quad (4)$$

$$\dot{\sigma}_{22}^{(n)} = -i\Omega_0 (\sigma_{12}^{(n)} - \sigma_{21}^{(n)}), \quad (5)$$

$$\dot{\sigma}_{12}^{(n)} = i(E_2 - E_1) \sigma_{12}^{(n)} + i\Omega_0 (\sigma_{11}^{(n)} - \sigma_{22}^{(n)}) - (1/2) D_1 \sigma_{12}^{(n)}. \quad (6)$$

Here  $\sigma_{11}^{(n)}(t)$ ,  $\sigma_{22}^{(n)}(t)$  are the probabilities of finding the left or the right dot occupied, with  $n$  electrons in the collector.  $\sigma_{12}^{(n)}(t)$  is the corresponding off-diagonal density-matrix element. Eqs. (4-6) allows detailed microscopic study of the measurement process. For instance, the influence of the detector on the measured system is determined by tracing out the detector states  $n$ . One finds

$$\dot{\sigma}_{11} = i\Omega_0 (\sigma_{12} - \sigma_{21}), \quad (7)$$

$$\dot{\sigma}_{12} = i(E_2 - E_1) \sigma_{12} + i\Omega_0 (2\sigma_{11} - 1) - (1/2) D_1 \sigma_{12}. \quad (8)$$

where  $\sigma_{ij} = \sum_n \sigma_{ij}^{(n)}$ , and  $\sigma_{22} = 1 - \sigma_{11}$ . As expected, the electron oscillations inside the double-dot are damped via the last (decoherence) term in Eq. (8), generated by the detector. Then the reduced electron density-matrix  $\sigma_{ij}(t)$  becomes the statistical mixture for  $t \rightarrow \infty$ :

$$\sigma(t) = \begin{pmatrix} \sigma_{11}(t) & \sigma_{12}(t) \\ \sigma_{21}(t) & \sigma_{22}(t) \end{pmatrix} \rightarrow \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \quad \text{for } t \gg t_0, \quad (9)$$

Yet, the relaxation time  $t_0$  *increases* with the dephasing rate  $D_1$ . It follows from Eqs. (7)-(8) that  $t_0 \simeq D_1/8\Omega_0^2$ , so that the continuous measurement slows down the transition rate between different states of the observed system. This result looks as a manifestation of the measurement collapse (Zeno effect). Yet, it was obtained from the continuous Schrödinger evolution of the entire system without any explicit relation to the collapse (cf. [3]).

Nevertheless, the problem arises with evaluation of the detector current. Consider for instance, the case when the electron density-matrix becomes the statistical mixture, Eq. (9). On the first sight one can expect that the detector current would display the average value,  $I_1/2$ . On the other hand, the mixture means that the electron actually occupies one of the dots, and therefore the detector should show either the current  $I_1$  or 0, but not the average. In fact, the behavior of the detector current cannot be determined from the reduced electron density-matrix, Eq. (9). We have to study the total density-matrix  $\sigma_{ij}^{(n)}$ , Eqs. (4)-(6), which provide quantum-mechanical description of the entire system, including the detector [4]. First consider the case of  $\Omega_0 = 0$ , so that the electron is permanently localized in the left well. Solving Eq. (4) by using the Fourier transform [5] we find

$$\sigma_{11}^{(n)}(t) = \bar{\sigma}^{(n)}(t) \simeq 1/(2\pi D_1 t)^{1/2} \exp \left[ -(D_1 t - n)^2 / 2 D_1 t \right] \quad (10)$$

where  $\bar{\sigma}^{(n)}(t)$  is the probability of finding  $n$  electrons in the collector. The number  $n$  increases with the rate  $D_1$  that determines the detector current. Now we are going to general case of  $\Omega_0 \neq 0$ . Consider again  $D_1 \gg \Omega_0$ , so that the electron, initially localized in one of the well stays there for a long time ( $t_0$ ). Solving numerically Eqs. (4)-(6) with the initial conditions  $\sigma_{11}^{(0)}(0) = 1$ ,  $\sigma_{12}^{(0)}(0) = \sigma_{22}^{(0)}(0) = 0$ , we find that  $\sigma_{11}^{(n)}(t) \simeq \bar{\sigma}^{(n)}(t)$  for  $t \lesssim t_0$  and  $\sigma_{12}^{(n)}(t)$  and  $\sigma_{22}^{(n)}(t)$  are very small. Therefore the detector behaves in the same way as if the electron stays localized in the left well (Zeno effect). A similar behavior would be obtained by introducing the measurement collapse. However, the situation is different when the electron is initially in the statistical mixture, or in the linear superposition. For instance, solving Eqs. (4)-(6)

for the initial conditions,  $\sigma_{11}^{(0)}(0) = \sigma_{22}^{(0)}(0) = 1/2$  and  $\sigma_{22}^{(0)}(0) = 1/2$  (or  $\sigma_{22}^{(0)}(0) = 0$ ), we obtain that  $\sigma_{11}^{(n)}(t) \simeq (1/2)\bar{\sigma}^{(n)}(t)$ , and  $\sigma_{22}^{(0)}(t) \simeq 1/2$  for  $t \lesssim t_0$ . It means that at  $t \simeq n/D_1$  one can find  $n$  or zero electrons in the collector with the same probability  $1/2$ . The first possibility implies that the left dot is occupied, and the second one corresponds to the occupied right dot. By assuming that one of these possibilities is actually realized, one finds the whole system starting its evolution from the new initial condition (collapse). Then the detector current would display quantum jumps, shown in Fig. 2. If however, the Schrödinger evolution is not interrupted, the detector would show the current either  $I_1$  or 0 with the probability  $1/2$ . This means the “telegraphic” noise for  $t \lesssim t_0$ . Such a behaviour is different from the recent result [?], based on continuous Schrödinger evolution. For  $t \gg t_0$ , however, Eqs. (4-6) give the average value  $I = I_1/2$  for the detector current, Fig. 2.

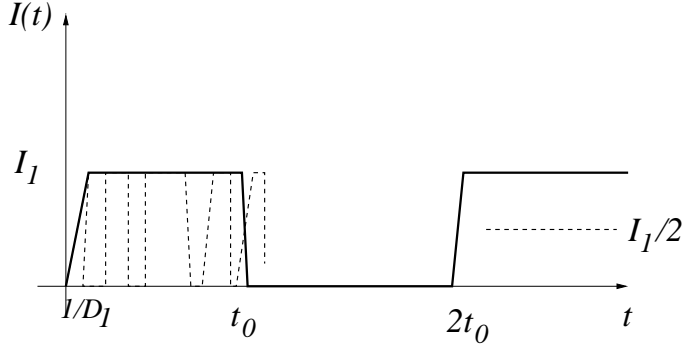


Fig. 2. Detector current as a function of time by assuming the collapse (the solid line) and without the collapse (the dashed line). The electron is initially in the statistical mixture or in the linear superposition.

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